A Study on Finite Difference Methods and Their Application to Heat Conduction Problems

ICTU Student 5
Thai Nguyen University of Information and Communication Technology
Thai Nguyen University, Thai Nguyen, Vietnam

***Abstract*—This thesis explores the application of finite difference methods to solve heat conduction problems. The author implemented and tested two finite difference methods in MATLAB: the explicit and implicit finite difference schemes. The explicit method was used to solve one-dimensional heat conduction problems, while the implicit method was applied to both one-dimensional and two-dimensional steady-state heat conduction problems. The author developed MATLAB programs for solving one-dimensional and two-dimensional steady-state heat conduction problems, as well as a program for solving multi-dimensional heat conduction problems using an alternating direction implicit method. The programs were tested with various boundary conditions and initial values, and the results were visualized using MATLAB's mesh function. The thesis also discusses the advantages and limitations of the explicit and implicit finite difference methods, highlighting the stability issues of the explicit method and the computational resources required by the implicit method. The author outlines future research directions, such as the use of computational methods to save computational resources and the development of programs for solving heat conduction problems in non-rectangular domains.**

***Index Terms*—finite difference method, heat conduction, MATLAB, explicit method, implicit method**

I. Introduction

The thesis begins with an introduction to MATLAB, covering its basic features and functions. The author explains the importance of MATLAB in numerical computations and its role in solving partial differential equations. The introduction then delves into the finite difference method, discussing its application to solving initial and boundary value problems for heat conduction equations. The author provides a detailed explanation of the finite difference method, including the derivation of finite difference approximations for derivatives and the stability analysis of the methods. The introduction also includes a discussion on the limitations of the finite difference method, such as the need for a fine grid to achieve accurate results and the computational complexity of solving large systems of equations. [1]

II. Methods

The author implemented two finite difference methods in MATLAB: the explicit and implicit finite difference schemes. The explicit method was used to solve one-dimensional heat conduction problems, while the implicit method was applied to both one-dimensional and two-dimensional steady-state heat conduction problems. The explicit method involves solving the heat equation at each time step using the values from the previous time step, while the implicit method requires solving a system of linear equations at each time step. The author developed MATLAB programs for these methods, including the necessary functions for setting up the grid, computing the finite difference approximations, and solving the resulting linear systems. The programs were tested with various boundary conditions and initial values, and the results were visualized using MATLAB's mesh function. [1]

Supervisor: Nguyễn Đình Dũng, Thai Nguyen University of Information and Communication Technology (nddung@ictu.edu.vn).

III. Results

The author successfully implemented MATLAB programs for solving one-dimensional and two-dimensional steady-state heat conduction problems using both explicit and implicit finite difference methods. The programs were tested with various boundary conditions and initial values, and the results were visualized using MATLAB's mesh function. The explicit method was used to solve one-dimensional heat conduction problems, and the implicit method was applied to both one-dimensional and two-dimensional steady-state heat conduction problems. The author also developed a program for solving multi-dimensional heat conduction problems using an alternating direction implicit method. The results showed that the explicit method was simpler to implement but could be unstable for certain problems, while the implicit method was more stable but required more computational resources. The author also reported the maximum absolute error between the numerical solution and the exact solution for each method, which varied depending on the problem and the grid size. [1]

IV. Discussion

The author discusses the advantages and limitations of the explicit and implicit finite difference methods. The explicit method is simpler to implement but can be unstable for certain problems, such as when the time step is too large. The author provides examples where the explicit method failed to converge, highlighting the importance of choosing an appropriate time step. The implicit method, on the other hand, is more stable but requires solving a system of linear equations at each time step, which can be computationally expensive. The author also discusses the use of iterative methods to solve the linear systems, such as the Gauss-Seidel method. The author concludes that the choice of method depends on the

specific problem and the available computational resources. [1]

V. Conclusion

The thesis concludes with a summary of the implemented methods and their successful application to heat conduction problems. The author successfully developed MATLAB programs for solving one-dimensional and two-dimensional steady-state heat conduction problems using both explicit and implicit finite difference methods. The explicit method was used to solve one-dimensional heat conduction problems, while the implicit method was applied to both one-dimensional and two-dimensional steady-state heat conduction problems. The author also developed a program using the Seidel relaxation method to solve two-dimensional steady-state heat conduction problems in rectangular domains. The implicit method was found to be more stable but required solving a system of linear equations at each time step. Future research could focus on improving the efficiency of the implicit method and developing more advanced numerical methods for solving heat conduction problems, such as using computational methods to save computational resources and developing programs for solving heat conduction problems in non-rectangular domains. [1]

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